Intonation and Tuning

Please note that there is a bug in Quicktime 5 which prevents intonations other than equal temperament from working. If you want to work with intonations or use the Tune directive in abcs you must revert to Quicktime 4. Quicktime 4.0.3 is still available from Apple as it's the last Quicktime version which works on the older machines with 680x0 processors.

Since time immemorial, musicians have tuned their instruments by ear, simply choosing the pitches which sound right for the tune and the key in which they are going to play. It's still the best way. However, musical instrument makers have a more difficult problem to deal with, since in many cases they need to fix the pitches which an instrument plays in advance, without knowing what the player will use it for. This is not a trivial problem, and a number of theoretical and practical solutions have been developed to deal with it.

The Pythagorean scale

The earliest mathematical theory of tuning was developed by the Greek mathematician Pythagoras (he of the square on the hypotenuse etc.). None of Pythagoras' own writings survive, and we only know about him from the writings of others; it seems that he attached great mystical significance to the number five, and spent some time developing a theory of life, the universe and everything based on five-fold symmetry. He probably used a monochord to investigate musical pitch, an instrument with one string and a sliding bridge which can be moved to any position. He will have noticed that if the bridge is placed so that the lengths of string on either side are in the ratio 2 : 3 they form the sweetest sounding of chords when played together. Now 2 + 3 = 5, and if you play a scale starting on the lower note you find that the higher note corresponds to the fifth note of that scale (the interval is what we now call a perfect fifth). Pythagoras was sure that his magic number was at the root of all music, and invented a way of tuning such that all notes of the scale were related to the tonic by way of the perfect fifth.

When we discuss the pitch of notes today, we make measurements of the frequency in cycles per second or hertz (Hz). Of course Pythagoras had no such concept - instead he would have thought in terms of the length of string necessary to sound a particular note. Since the frequency produced by a string is inversely proportional to its length, we can say that the that the string length ratio (2/3) which he observed corresponds to a frequency ratio of 3/2 i.e. the shorter string is vibrating exactly one and a half times faster. To construct a pythagorean scale, we start with a given tonic (say A = 440.0 Hz, since that is our modern pitch standard), multiply by three and divide by two to get the pitch of the E above:

A = 440.0
E = 440.0 * 3/2 = 660.0

To get the next note, B, we multiply this again by 3/2, but this takes us out of the octave, so we divide by an extra two to get us back in the same octave:

B = 660 * 3/4 = 495.0

If we carry this process on for long enough, multiplying by three and dividing by two or four as necessary to stay in the same octave, we will generate frequencies for all twelve notes of the chromatic scale:

A	440.0
A#	469.9
B	495.0
C	528.6
C#	556.9
D	594.7
D#	626.5

E   660.0
F   704.8
F#   742.5
G   792.9
G#   835.3

This is fine as far as it goes; the problem with it becomes apparent when you calculate the next frequency, the octave A'. The last note calculated above is D = 594.7, so A' = 594.7 * 3/2 = 892.0 which is quite wrong, as the octave should exactly double the frequency to 880.0. This will not have seriously bothered Pythagoras, since ancient Greek music was limited to less than an octave in range, and his lyre probably only had seven strings. The Pythagorean scale is not circular - that is, it doesn't get you back to where you started. The reason for this is that twelve fifths does not exactly add up to seven octaves, and the difference is known as the Comma of Pythagoras.
Simple tunes can sound very sweet in this intonation (all the fifths are perfect after all). All the pitches are sharp compared with modern (equal) temperament, with the minor third, fourth and minor seventh particularly noticeable.

Just Intonation

It is clear that musicians who can choose the pitch of every note that they play (such as singers and violinists) have no problem with any of this. As long as they play unaccompanied they can produce music which is perfectly in tune in any key and any mode. So what is the scale in which they play, and why does it sound so good? Using physical measuring equipment or computer programs it is easy to demonstrate that most sounds consist of multiple frequencies. In musical sounds these frequencies have a regular mathematical relationship to the fundamental frequency which we hear as the pitch of the note. In addition to the fundamental frequency, we find a series of overtones or harmonics, consisting of frequencies at twice, three times, four times (etc.) the fundamental frequency. There is a little confusion as to the numbering of these - the frequency which is twice that of the fundamental is called the second harmonic, or the first overtone. I'm going to call them harmonics and give them the number which is the multiple of the fundamental frequency which they represent, so when I say "seventh harmonic" I'm talking about a frequency seven times that of the fundamental. So, our note of A = 440.0 can also be expected to contain frequencies of:

880.0 (2nd harmonic)
1320.0 (3rd harmonic)
1760.0 (4th harmonic)

and so on.

It is the proportions of these harmonics which determine the timbre of the note, and depending on the instrument and pitch, these may extend up to the twelfth or even higher harmonic. (Sounds which consist of component frequencies which do not relate to this harmonic series are not perceived as having musical pitch - a dusbin lid hit with a hammer produces a very complex mixture of frequencies, but since they mostly don't relate to the harmonic series we don't hear the result as having a musical pitch.)

When we hear two musical notes played (either together or consecutively) our perception of the interval between them depends on how the harmonics interact with one another. For example, in the case of the perfect fifth, if we play an A 440 with an E 660, the third harmonic of the A (440 * 3 = 1320) matches exactly the second harmonic of the E (660 * 2 = 1320). In fact, every harmonic of the lower note which is a multiple of three matches an even-numbered harmonic of the higher note. There are similar matches to be found for other intervals - for example if we tune the C# to a frequency 5/4 times the fundamental A, its fourth harmonic (and all multiples of four) will match multiples of the fifth harmonic of the fundamental. It seems that to take advantage of this effect, all we have to do is to tune each note of the scale to a simple ratio of the fundamental frequency, and this is the basis of Just Intonation:

A   440.0
A#   440 * 16/15 = 469.3
B   440 * 9/8 = 495.0

C 440 * 6/5 = 528.0
C# 440 * 5/4 = 550.0
D 440 * 4/3 = 586.7
D# 440 * 7/5 = 616.0 (or 440*10/7 = 628.6)
E 440 * 3/2 = 660.0
F 440 * 8/5 = 704.0
F# 440 * 5/3 = 733.3
G 440 * 16/9 = 782.2
G# 440 * 15/8 = 825.0
A 440 * 2 = 880.0

This sounds quite lovely, provided that you are playing in the key of A. In other keys, however, the effect can be disastrous. For example, if you play in the key of B major (495 Hz), the third is 616 giving a ratio of 616/495, or 1.24 instead of 1.25, and the fifth is 733.3 giving a ratio of 733.3/495, or 1.48 instead of 1.5. This is not a good way to tune an instrument which is to be played in different keys. Of course if your instrument is a computer program it can work much better, since the program can recalculate all the pitches when you play in a different key, and this is what BarFly does.

The 19th century scientists Delezenne and von Helmholtz both performed experiments which demonstrated that an unaccompanied violinist chooses pitches (especially the third and sixth) which are close to those predicted above.

Equal Temperament.

The modern solution to the problem is to tune so that all the semitones are equal, and equal to one twelfth of an octave. All keys are now exactly the same, and all intervals except the octave are very slightly out of tune. Mostly we don't notice, and we can play music which modulates into any key. Effectively the Comma of Pythagoras has been distributed amongst all the notes so that it is no longer noticeable. To calculate the pitches, we multiply each note by the twelfth root of two (2^(1/12) = 1.05946309) to get the next semitone:

A 440.0
A# 440.0 * 1.05946309 = 466.2
B 466.2 * 1.05946309 = 493.9
C 493.9 * 1.05946309 = 523.3
C# 523.3 * 1.05946309 = 554.4
D 554.4 * 1.05946309 = 587.3
D# 587.3 * 1.05946309 = 622.3
E 622.3 * 1.05946309 = 659.3
F 659.3 * 1.05946309 = 698.5
F# 698.5 * 1.05946309 = 740.0
G 740.0 * 1.05946309 = 784.0
G# 784.0 * 1.05946309 = 830.6
A 830.6 * 1.05946309 = 880.0

Equal temperament is the intonation which Quicktime (and BarFly) uses by default.

Highland Bagpipe.

The bagpipe intonation supplied with BarFly is that given for the MacDonald chanter in the book "Bagpipes and Tunings" by Theodor Podnos. This is an arbitrary tuning - it's the traditional way in which Scottish war pipes are tuned, and varies a little from one instrument to another. Unlike the other intonations, this one is rooted on the key of A, and the pitches will remain the same for tunes in different keys. The main characteristics are that the C# and F# are tuned very flat. The notes which are not present on the bagpipe chanter are left in equal temperament. One other peculiarity of highland bagpipe tuning is that it is tuned to a different pitch standard to that used by the rest of the musical world. Rather than use the modern concert pitch (A = 440.0) highland pipes use the older Queen's Hall pitch standard (A = 461.6), which is close to a semitone sharp of the modern pitch. (You will occasionally hear it said that the pipes are in Bb for this reason.) For the sake of comparison with the other scales, the pitches here are given relative to concert pitch A.

G 394.0

A 440

B 493.0

C# 535.8
D 585.6

E 660.4

F# 720.2
G 788.1

A 880

The Intonation Editor.

You can change the existing intonations and create new ones using the Edit IntonationsÉ command from the Play menu. This leads to a dialog with a popup menu at the top in which you can select an intonation to edit. The checkbox below this determines whether the intonation is rooted on a particular note, or tunes itself to the key specified in the K: field of the tune being played. Eleven text boxes are for entering the pitch settings for each note of the scale. Pitch is specified as a ratio relative to the tonic, and must be in the range 1.0 < ratio < 2.0. You can enter the numbers either as decimals (e.g. 1.5) or as a fraction (3/2). The Save button makes the change permanent (intonation settings are stored in the BarFly Preferences file). Intonations are applied to the notes when played as pitch bend values, and Quicktime measures pitch bend in units of 1/256 semitone. The ratios which you enter are converted into these units before storage. You may notice that if you enter values here and save them they may not be exactly the same the next time you open the dialog. (e.g. you may have entered the ratio 1.2, but it appears as 1.1999 the next time you look - this is not an error, it's just that the ratio has been rounded to the nearest 1/256 semitone.)

Note that:

¥ You cannot edit "Equal Temperament" because it is fundamental to the working of the program. However, when you create a new intonation the default values which the program fills in are equal temperament, so you can always work on a copy.

¥ While you can delete the three temperaments supplied, it will all be to no avail, as the program will re-create them the next time it is started. This is useful if you mess up one of the supplied temperaments and want to get back to the original settings.

¥ The Save and Delete buttons take action immediately and there is no Undo available. The Cancel button just dismisses the dialog.

Play Note

This command lets you set a note (or a chord) playing continuously for tuning purposes, or for use as a drone. Enter the note in the dialog in abc format (i.e. if you want C sharp, write ^C, and add commas or apostrophes to change the octave). If you need a note "in the cracks" you can enter a pitch bend value in units of ±1/256 semitone. When you click the Play button the program will display the frequency and set the note playing. You can now enter a new value and click Play again if you want a chord. Clicking on the Cancel button dismisses the dialog leaving the notes playing - you can silence them either by returning to the dialog and hitting the Stop button or by simply typing a Command-period.

This command only works with Quicktime Instruments, and you should choose a suitable instrument (one with an infinite decay time) before using the command.

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